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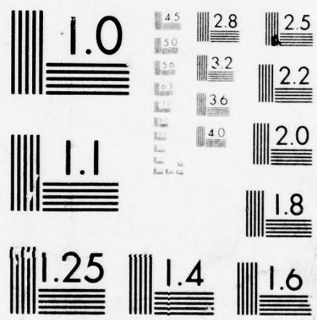

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EFFECT OF DAMPING AT THE SUPPORT OF A ROTATING BEAM ON VIBRATIONS

J.D. Vasilakis
J.J. Wu

October 1977

US ARMY ARRADCOM

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
LARGE CALIBER WEAPON SYSTEM LABORATORY
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The paper presents a formulation for the study of damping effects in dynamic structural problems and a specific application. A finite element formulation is first derived from the versatile unconstrained variational approach. The vibration of a rotating beam is used here as a concrete example. Viscous damping terms at the support can be present due to either local deflection or rotation. These terms can obviously affect the frequencies of the rotating beam. They are easily incorporated in the present formulation using the concept of unconstrained (See Reverse)

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variations. Numerical data will be presented to demonstrate the qualitative as well as quantitative effects on the vibratory behavior of this rotating beam due to such damping terms.

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1. INTRODUCTION. The applicability of the unconstrained adjoint variational statement in solving nonconservative stability problems has been shown in a series of articles [1-4]. The problems involved the stability of beams or columns subject to concentrated or distributed tangential loads. The problems are solved by finding the variational statement associated with the differential equation and they are rendered unconstrained by incorporating the geometric boundary conditions into the variational statement through the use of Lagrange multipliers. With the variational statement now available, the problem is discretized and solved using finite elements. Various types of external forces and geometric boundary conditions can be handled using the above techniques. It is the purpose of this paper to incorporate into the above-mentioned formulation the effect of support damping and to examine its effect on the solution. The specific problem chosen was that of a rotating cantilever beam of constant cross section.

This problem was chosen for its application to a simplified helicopter blade and although no nonconservative forces are considered the solution technique outlined above is applicable.

The effects of support damping on the vibration response of beams has been investigated by others. Fu and Mentel [5] and Mentel [6] considered support damping due to viscoelastic layers applied to the ends of a beam in its supports. The effect of translational (axial) damping was found to be of the same order in terms of energy dissipation at the supports as that of material damping. Material damping effects were found to stiffen the beam which increased both, the resonance frequency and the energy dissipation at the supports. They also found that rotational motion dominates the damping properties if all parameters are suitably optimized. Ruzicka [7] presented an evaluation of the resonance characteristics of undirectional vibration isolation systems including directly coupled (Kelvin/Voight) and elastically coupled (Zener) damping elements. His results were mostly for the Zener model and he found that resonant frequencies of vibration isolation systems with viscous damping may increase or decrease with an increase in the viscous damping coefficients depending on the stiffnesses in the system. MacBain and Genin treated support flexibility in a series of papers. Support and material damping was introduced in [8]. The support is viewed as a complex rotational support stiffness based on bounds for the elastic modulus found in their earlier papers. They find that when the support damping constant is an optimum, the support loss factor is also an optimum, and system loss factor reaches a maximum value. This same value of the support loss factor is also that which critically dampens the system in free vibration.

The results presented here show the effects of support damping on the flexural frequencies of vibration of the rotating beam with both deflection and rotation flexibility at the support.

2. PROBLEM STATEMENT. The geometry of the problem is shown in Figure 1. The beam has a constant cross section of area A, density ρ, Young's modulus, E, and moment of inertion, I. The beam rotates about an axis fixed at one end of the beam and is flexibly supported at that end by a deflection spring, k_1, and a rotation spring, k_2. Viscous dashpots, c_1 and c_2, are assumed in parallel to the deflection and rotation springs, respectively. The beam rotates at constant angular velocity, Ω. S(0) represents support reaction.

The differential equation governing the motion is given by [9]

$$u'''' - \frac{\Omega^2 \rho A}{2EI}\left[(\ell^2 - x^2)u'\right]' + \frac{\rho A}{EI}\ddot{u} = 0 \tag{1}$$

and the boundary conditions are

at x = 0

$$u''(0) - \frac{c_2}{EI}\dot{u}'(0) - \frac{k_2}{EI}u'(0) = 0$$

$$u'''(0) + \frac{c_1}{EI}\dot{u}(0) + \frac{k_1}{EI}u(0) - \frac{S(0)}{EI}u'(0) = 0 \tag{2}$$

at x = ℓ

$$u''(\ell) = 0$$

$$u'''(\ell) = 0 \tag{3}$$

The differential equation and boundary conditions are rewritten using dimensionless variables and parameters defined by the following:

$$\bar{u} = \frac{u}{\ell}, \quad \bar{x} = \frac{x}{\ell}, \quad Q = \frac{\Omega^2 \rho A \ell^4}{2EI}, \quad \bar{t} = \left[\frac{EI}{\rho A \ell^4}\right]^{1/2} t$$

$$\bar{c}_1 = \frac{c_1 \ell}{(EI\rho A)^{1/2}} \qquad \bar{c}_2 = \frac{c_2}{(EI\rho A)^{1/2}\ell} \tag{4}$$

$$\bar{k}_1 = \frac{k_1 \ell^3}{EI} \qquad \bar{k}_2 = \frac{k_2 \ell}{EI}$$

Time is removed by assuming displacements to have the form

$$\bar{u}(\bar{x},\bar{t}) = \bar{u}(\bar{x})e^{\lambda \bar{t}} \tag{5}$$

Then the differential equation becomes (dropping the bar symbol):

$$u'''' - Q[(1 - x^2)u']' + \lambda^2 u = 0 \tag{6}$$

and the boundary conditions

$$x = 0 \begin{cases} u''(0) - (\lambda c_2 + k_2)u'(0) = 0 \\ u'''(0) + (\lambda c_1 + k_1)u(0) - Qu'(0) = 0 \end{cases} \tag{7}$$

$$x = 1 \begin{cases} u''(1) = 0 \\ u'''(1) = 0 \end{cases} \tag{8}$$

The eigenvalues, λ, will be complex,

$$\lambda = \lambda_R + i\lambda_I \tag{9}$$

The frequencies will be given by

$$\omega = \lambda_I \left[\frac{EI}{\rho A \ell^4}\right]^{1/2} \tag{10}$$

and for this problem, $\lambda_R < 0$, i.e., the real component of the eigenvalue is negative and no instabilities should exist.

3. VARIATIONAL STATEMENT. To find the form of the variational statement, the differential equation is multiplied by an arbitrary variation of the adjoint field variable, $\delta v(x)$, and integrated over the beam length. Integration by parts indicates the form of the variational statement and the natural boundary conditions. The geometric boundary conditions are attached with the values of the springs and dashpots playing the role of Lagrange multipliers. The variational statement is finally given by

$$\delta J = 0 \tag{11}$$

where

$$J = \int_0^1 [u''v'' + Q[1 - x^2]u'v' + \lambda^2 uv]dx +$$

$$+ (\lambda c_1 + k_1)u(0)v(0) + (\lambda c_2 + k_2)u'(0)v'(0) \tag{12}$$

Performing the variation of J with respect to u and v, one can arrive at the original boundary value problem as well as its adjoint. In this case the two problems are identical.

4. FINITE ELEMENTS. To solve the problem using finite element techniques, the beam must be divided into segments and the nodes defined. The value for the unknown variable within each element must then be expressed in terms of the nodal values of the function through the use of interpolating shape functions. A global expression, or matrix is then formed and the eigenvalues found.

The procedure begins by taking the variation of Equation (12) and allowing the variations in the problem variable, $\delta u(x)$, to be zero,

$$\int_0^1 [u''\delta v'' + Q(1 - x^2)u'\delta v' + \lambda^2 u\delta v]dx +$$

$$(\lambda c_1 + k_1)u(0)\delta v(0) + (\lambda c_2 + k_2)u'(0)\delta v'(0) = 0 \tag{13}$$

The beam is divided into L elements, letting

$$\xi = L \{x - \frac{i-1}{L}\} \qquad i = 1,2,3 \ldots L \tag{14}$$

be the running coordinate in each element. Substituting Eq. (14) into Eq. (13):

$$\sum_{i=1}^{L} \int_0^1 [L^3 u^{(i)''} \delta v^{(i)''} + \frac{Q}{L} (L^2 - [\xi + (i - 1)]^2) u^{(i)'} \delta v^{(i)'}$$

$$+ \frac{\lambda^2}{L} u^{(i)} \delta v^{(i)}] d\xi + (\lambda c_1 + k_1) u^{(1)}(0) \delta v^{(1)}(0) + (\lambda c_2 + k_2)$$

$$L^2 u^{(1)'}(0) \delta v^{(1)'}(0) = 0 \tag{15}$$

In order that the displacements and their derivatives within an element be expressed in terms of their nodal values, the coordinate vectors

$$\bar{U}^{(i)^T} = \{U_1^{(i)} \quad U_2^{(i)} \quad U_3^{(i)} \quad U_4^{(i)}\}$$

and (16)

$$\bar{V}^{(i)^T} = \{V_1^{(i)} \quad V_2^{(i)} \quad V_3^{(i)} \quad V_4^{(i)}\}$$

are introduced. $U_1^{(i)}$, $U_2^{(i)}$ represent the displacement and slope at the left end of the ith element and $U_3^{(i)}$ and $U_4^{(i)}$ represent deflection and slope at the right end. A similar interpretation is applied to the adjoint coordinate vector $\bar{V}^{(i)}$. The transform is indicated by T.

Hermitian polynomials are used to relate the displacements within an element to its nodal values, hence, the following shape function is assumed,

$$\bar{a}^T(\xi) = \{1 - 3\xi^2 + 2\xi^3 \quad \xi - 2\xi^2 + \xi^3 \quad 3\xi^2 - 2\xi^3 \quad -\xi^2 + \xi^3\} \tag{17}$$

So that

$$u^{(i)}(\xi) = \bar{a}^T(\xi)\bar{U}^{(i)}$$

$$v^{(i)}(\xi) = \bar{a}^T(\xi)\bar{V}^{(i)} \tag{18}$$

Substituting Eq. (18) into Eq. (15)

$$\sum_{i=1}^{L} \bar{U}^{(i)^T}\{L^3\bar{C} + [QL - \frac{Q}{L}(i-1)^2]\bar{B} - \frac{Q}{L}\bar{E} - 2(i-1)\frac{Q}{L}\bar{D} + \frac{\lambda^2}{L}\bar{A}\}\delta\bar{V}^{(i)}$$

$$+ (\lambda c_1 + k_1)\bar{U}^{(1)^T}\bar{H}\delta\bar{V}^{(1)} + L^2(\lambda c_2 + k_2)\bar{U}^{(1)^T}\bar{F}\delta\bar{V}^{(1)} = 0 \tag{19}$$

where

$$\bar{A} = \int_0^1 \bar{a}(\xi)\bar{a}^T(\xi)d\xi, \qquad \bar{E} = \int_0^1 \xi^2\bar{a}'(\xi)\bar{a}^{T'}(\xi)d\xi$$

$$\bar{B} = \int_0^1 \bar{a}'(\xi)\bar{a}^{T'}(\xi)d\xi, \qquad \bar{F} = \bar{a}'(0)\bar{a}'^T(0)$$

$$\bar{C} = \int_0^1 \bar{a}''(\xi)\bar{a}^{T''}(\xi)d\xi$$

$$\bar{D} = \int_0^1 \xi\,\bar{a}'(\xi)\bar{a}^{T'}(\xi)d\xi, \qquad \bar{H} = \bar{a}(0)\bar{a}^T(0) \tag{20}$$

Regrouping of (19),

$$\sum \bar{U}^{(i)^T}\{\lambda^2 P^{(i)} + \lambda R^{(i)} + S^{(i)}\}\delta\bar{V}^{(i)} = 0 \tag{21}$$

where

$$P^{(i)} = \bar{A}/L \qquad i = 1,2,\ldots L \tag{22}$$

$$R^{(1)} = + c_1\bar{H} + c_2\bar{F}L^2 \qquad i = 1$$

$$R^{(i)} = 0 \qquad i = 2,3,\ldots L \tag{23}$$

$$S^{(1)} = L^3\bar{C} + QL\bar{B} - Q/L\,\bar{E} + k_1\bar{H} + k_2\bar{F}L^2 \qquad i = 1$$

$$S^{(i)} = L^3\bar{C} + QL[1 - \frac{1}{L^2}(i-1)^2]\bar{B} - \frac{Q}{L}\bar{E} - 2(i-1)Q\bar{D}/L \qquad i = 2,3,\ldots L \tag{24}$$

Using certain continuity conditions between the element nodal values

$$U_1^{(i)} = U_3^{(i-1)} \qquad V_1^{(i)} = V_3^{(i-1)}$$
$$U_2^{(i)} = U_4^{(i-1)} \qquad V_2^{(i)} = V_4^{(i-1)} \tag{25}$$

One can write

$$\bar{U}^{(T)}\{\lambda^2[P] + \lambda[R] + [S]\}\delta V = 0 \tag{26}$$

where now

$$\bar{U}^{(T)} = \{U_1^{(1)} \quad U_2^{(1)} \quad U_3^{(1)} \quad U_4^{(1)} \quad U_3^{(2)} \quad U_4^{(2)} \ldots U_3^{(L)} \quad U_4^{(L)}\}$$

$$\bar{V}^{T} = \{V_1^{(1)} \quad V_2^{(1)} \quad V_3^{(1)} \quad V_4^{(1)} \quad V_3^{(2)} \quad V_4^{(2)} \ldots V_3^{(L)} \quad V_4^{(L)}\}$$

[P], [R], [S] are N x N matrices (N = 2L + 2). Since δV is arbitrary, the eigenvalue problem reduces to

$$\bar{U}^{(T)}\{\lambda^2[P] + \lambda[R] + [S] = 0 \tag{27}$$

for the eigenvalues of the problem.

An existing subroutine was used to find the eigenvalues which required the standard eigenvalue problem form

$$\{[A] + \lambda[I]\}\bar{U} = 0 \tag{28}$$

The equation

$$\{\lambda^2[A] + \lambda[B] + [C]\}\bar{U} = 0 \tag{29}$$

can be reduced to Eq. (28) by defining

$$\bar{W} = \lambda\bar{U} \tag{30}$$

This leads to the matrix equation

$$\left[\begin{array}{c|c} [0] & [I] \\ \hline -[A]^{-1}[C] & -[A]^{-1}[B] \end{array}\right] \left\{\begin{array}{c} \bar{U} \\ \hline \bar{W} \end{array}\right\} = \lambda \left\{\begin{array}{c} \bar{U} \\ \hline \bar{W} \end{array}\right\} \tag{31}$$

which is in the required format. The drawback here is that the order of the matrix has been doubled. Equation (31), however, is the form used for computing the eigenvalues.

5. RESULTS. Figures 2 and 3 show the effects for zero damping at the support. Figure 2 shows the effect of the rotation spring (k_2) only on the frequency with load as a parameter. The deflection spring is assumed to be infinitely stiff. For Q = 0, the beam is only vibrating and is not rotating. One can see that a stiffening effect occurs, i.e., the vibrating frequencies increase with an increase in the rotation spring constant. The frequencies rapidly approach those for a fully clamped vibrating and rotating beam. These results also fall within the bounds computed by Boyce, DiPrima and Handelman [10]. In Figure 3, the rotation spring is assumed to be infinitely stiff and the effect of varying the deflection spring is shown for different loads. Again, in general, there is a stiffening effect as the deflection spring value increases. For very small values of the deflection spring, the first vibrating frequency decreases slightly for increased loads, although only Q = 0 and 200 are shown. As k_1 is increased, there are cross over points after which higher loads do imply higher frequencies.

Figure 4 shows the effect of rotation damping at the support of a beam having rotation flexibility at the support. The deflection spring is assumed infinitely stiff and the deflection dashpot is zero. The figure is for a specific value of the rotation spring, $k_2 = 1$, and shows the first two eigenvalues for each of two loads, Q = 0 and Q = 100. A stiffening effect is found for increasing damping for $Q \neq 0$. For Q = 0, there is very slight decrease for very small damping values. For smaller rotation spring constants, and zero load frequencies decrease with increased damping as shown in Figure 4 by the portion of the results for $k_2 = .1$. These results are better shown in Figure 5. The results in Figure 4 are interesting since one would expect a decrease in frequency as damping is increased. Stiffening effects due to damping are found elsewhere [5] and could be due to the manner in which it is introduced in the problem. Figure 4 also shows that the results for the fully damped beam are approached rapidly as damping is increased. Figure 5 shows the vibrating frequencies on a complex plane for a beam with rotation flexibility and damping. The load is zero (non-rotating beam) and the rotation spring is kept at $k_2 = .1$ while the dashpot value changes. The arrows adjacent to the curves show the direction of the values on the curve as damping increases. For zero damping, the results are purely imaginary and are approximately .54 and 15.5 for the 1st and 2nd frequencies. As damping increases, the frequencies become complex with the imaginary components decreasing for the first eigenvalue and increasing for the second. The behavior of the first frequency is interesting. As the damping value increases, the imaginary component vanishes (as also seen in Figure 4) as if the system becomes critically damped. However, the real component can also be followed on the complex plot and the beam appears to vibrate again in this first mode as damping increases further. For sufficiently large damping the frequencies seem to approach those for a beam which is damped at the support. Points on the real axis represent zero motion but move with changes in damping values to points on the real axis where it is intersected by a branch or

mode. Figure 6 shows the same results for load Q = 25. A final result for rotation flexibility is shown in Figure 7 for $k_2 = 10$. Here the rotation spring is relatively stiff and the fully damped results are rapidly approached with initial effects for near zero dashpot values overshadowed.

The effect of support damping on the frequencies on beams with deflection flexibility are shown in Figures 8-10. A decrease in frequency with increased damping is seen here. Figures 9 and 10 show the effect on the complex plane for Q = 0 and Q = 200, respectively.

Finally, Figure 11 shows the results of a case when both rotation spring and deflection spring flexibilities are allowed. Little effect on frequency is noted as the rotation spring is varied while the deflection spring and dashpot remain constant in value. An almost parallel increase in frequencies are found when the rotation dashpot is increased in value. The investigation into the response of the beam with all springs and dashpots finite was limited to those shown. A study should be performed to indicate areas where the effects on beam response will be most pronounced.

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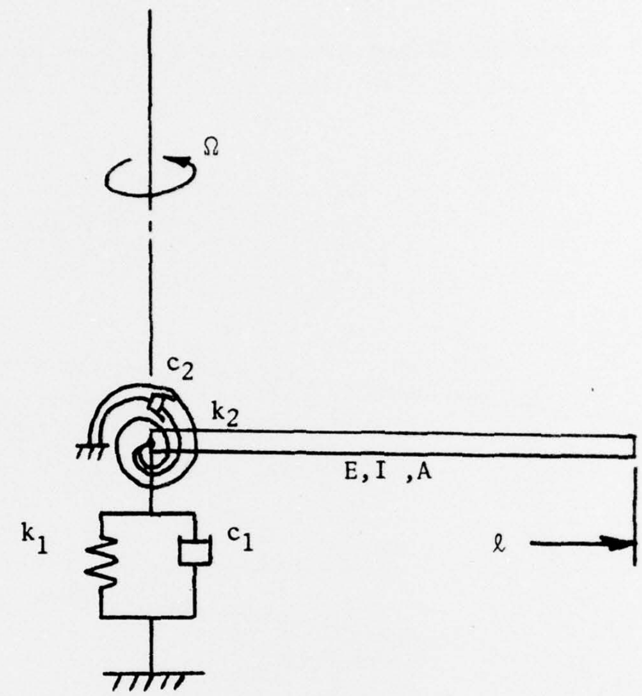


Figure 1. Geometry of Rotating Beam

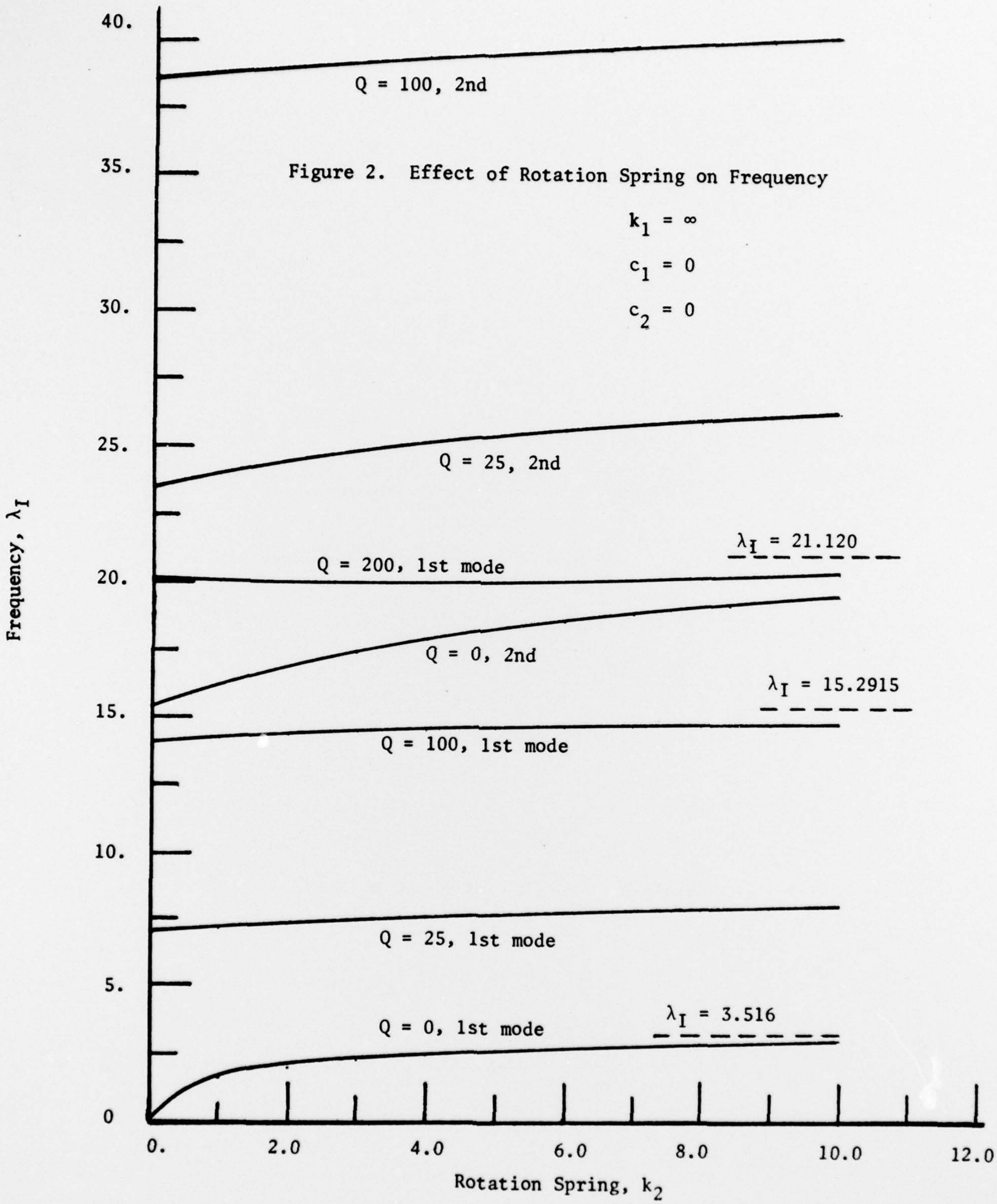


Figure 2. Effect of Rotation Spring on Frequency
$k_1 = \infty$
$c_1 = 0$
$c_2 = 0$
Q = 100, 2nd
Q = 25, 2nd
Q = 200, 1st mode
$\lambda_I = 21.120$
Q = 0, 2nd
$\lambda_I = 15.2915$
Q = 100, 1st mode
Q = 25, 1st mode
Q = 0, 1st mode
$\lambda_I = 3.516$
Frequency, λ_I
Rotation Spring, k_2
40.
35.
30.
25.
20.
15.
10.
5.
0
0.
2.0
4.0
6.0
8.0
10.0
12.0

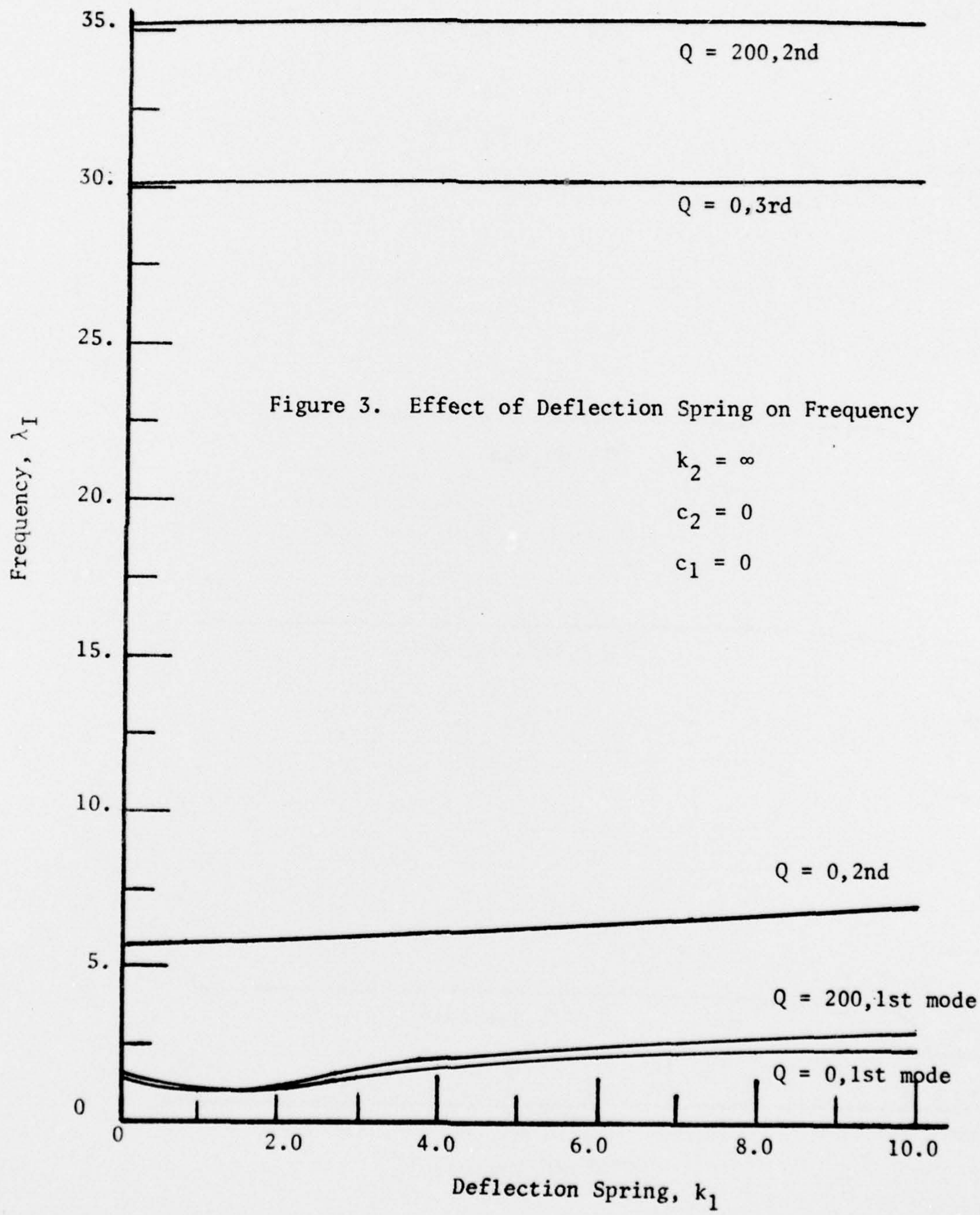


Figure 3. Effect of Deflection Spring on Frequency

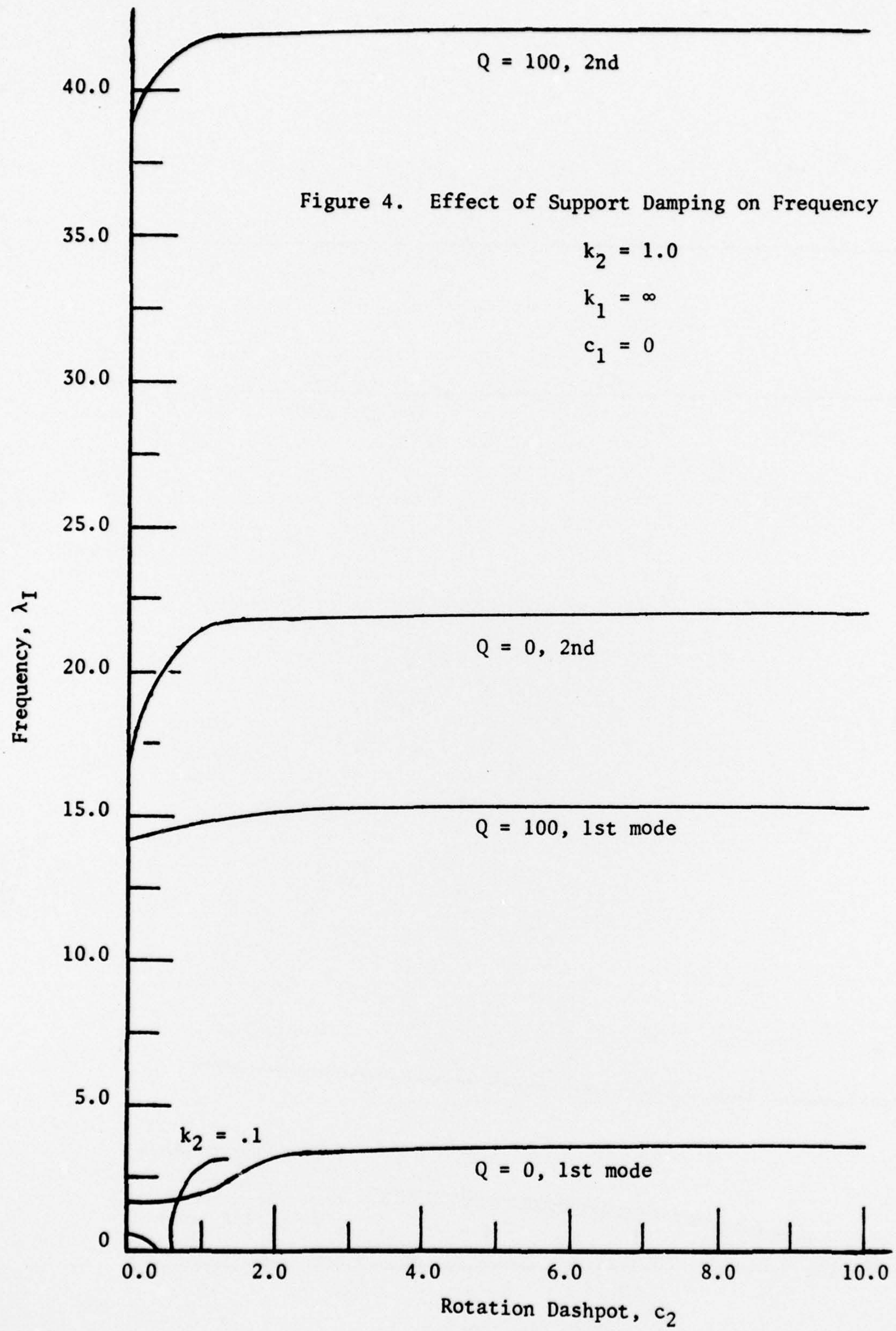


Figure 4. Effect of Support Damping on Frequency

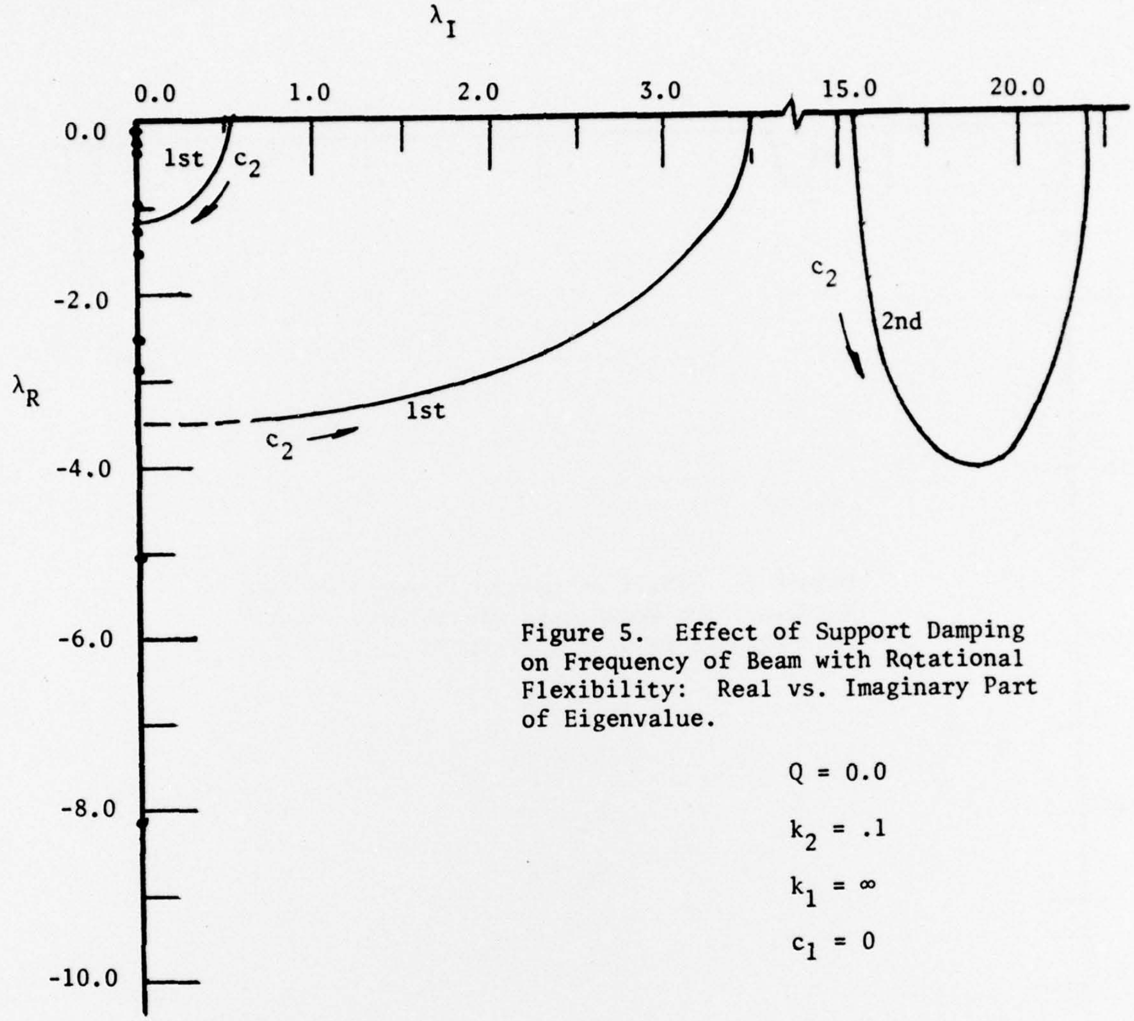


Figure 5. Effect of Support Damping on Frequency of Beam with Rotational Flexibility: Real vs. Imaginary Part of Eigenvalue.

$Q = 0.0$

$k_2 = .1$

$k_1 = \infty$

$c_1 = 0$

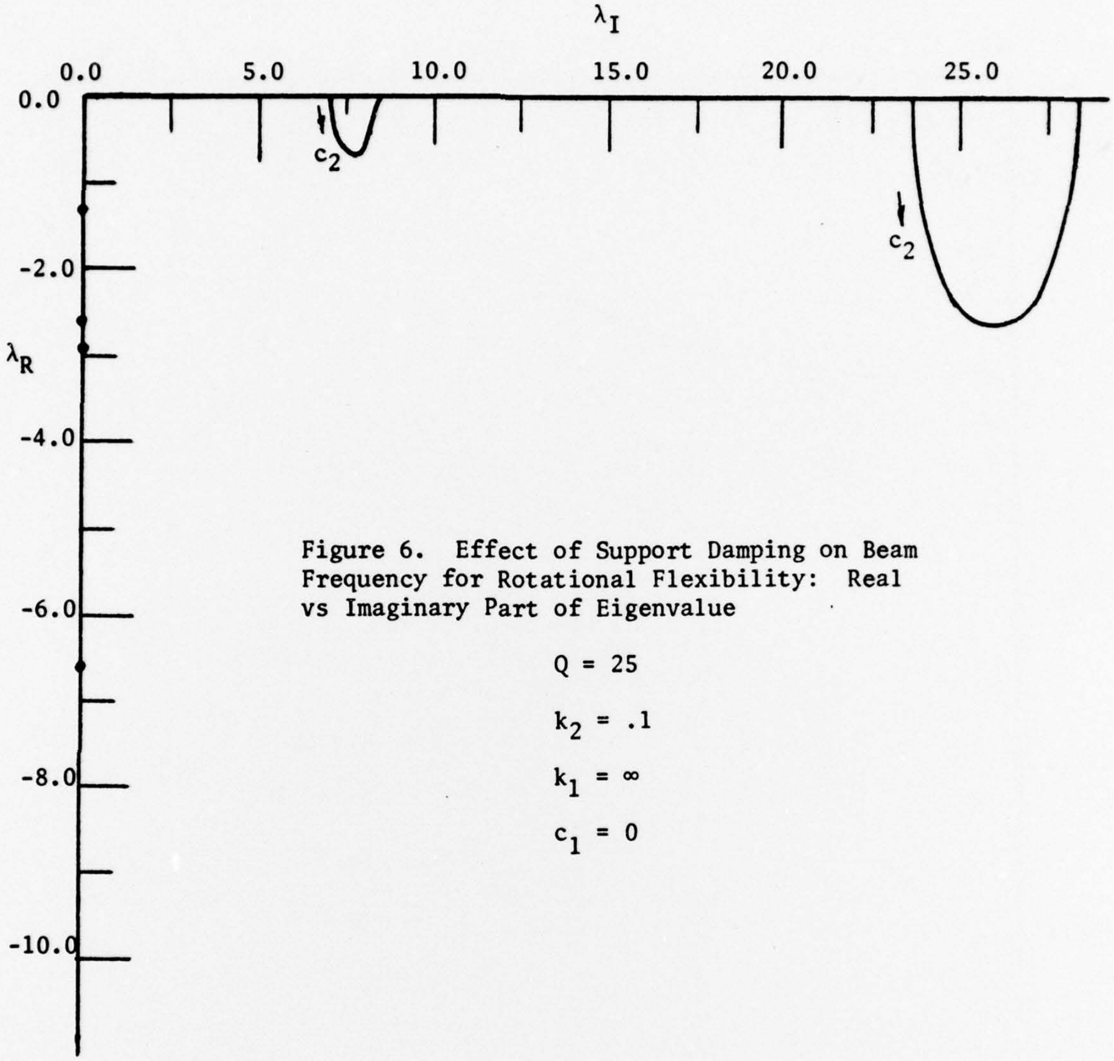


Figure 6. Effect of Support Damping on Beam Frequency for Rotational Flexibility: Real vs Imaginary Part of Eigenvalue

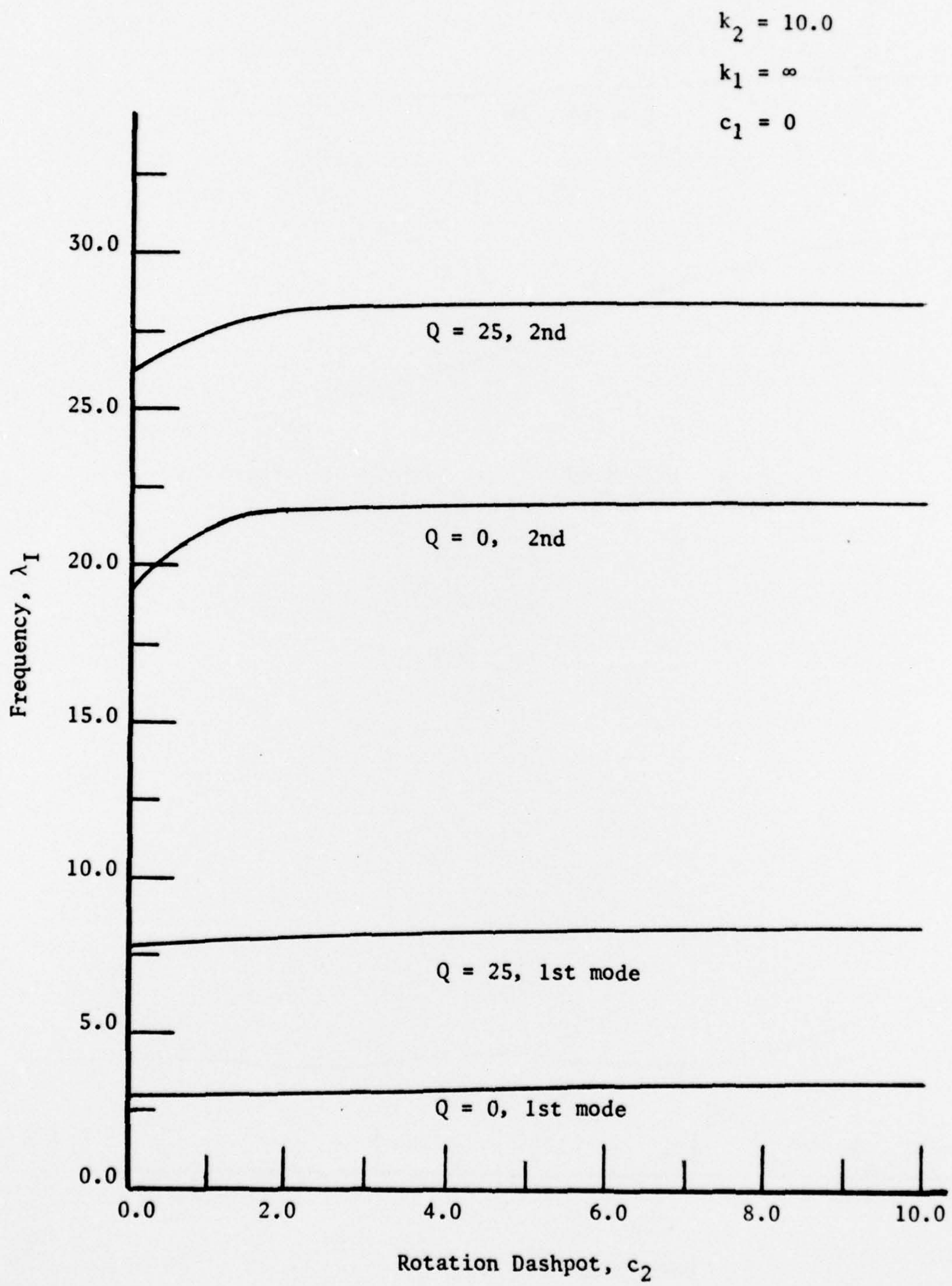
$Q = 25$

$k_2 = .1$

$k_1 = \infty$

$c_1 = 0$

Figure 7. Effect of Support Damping on Frequency



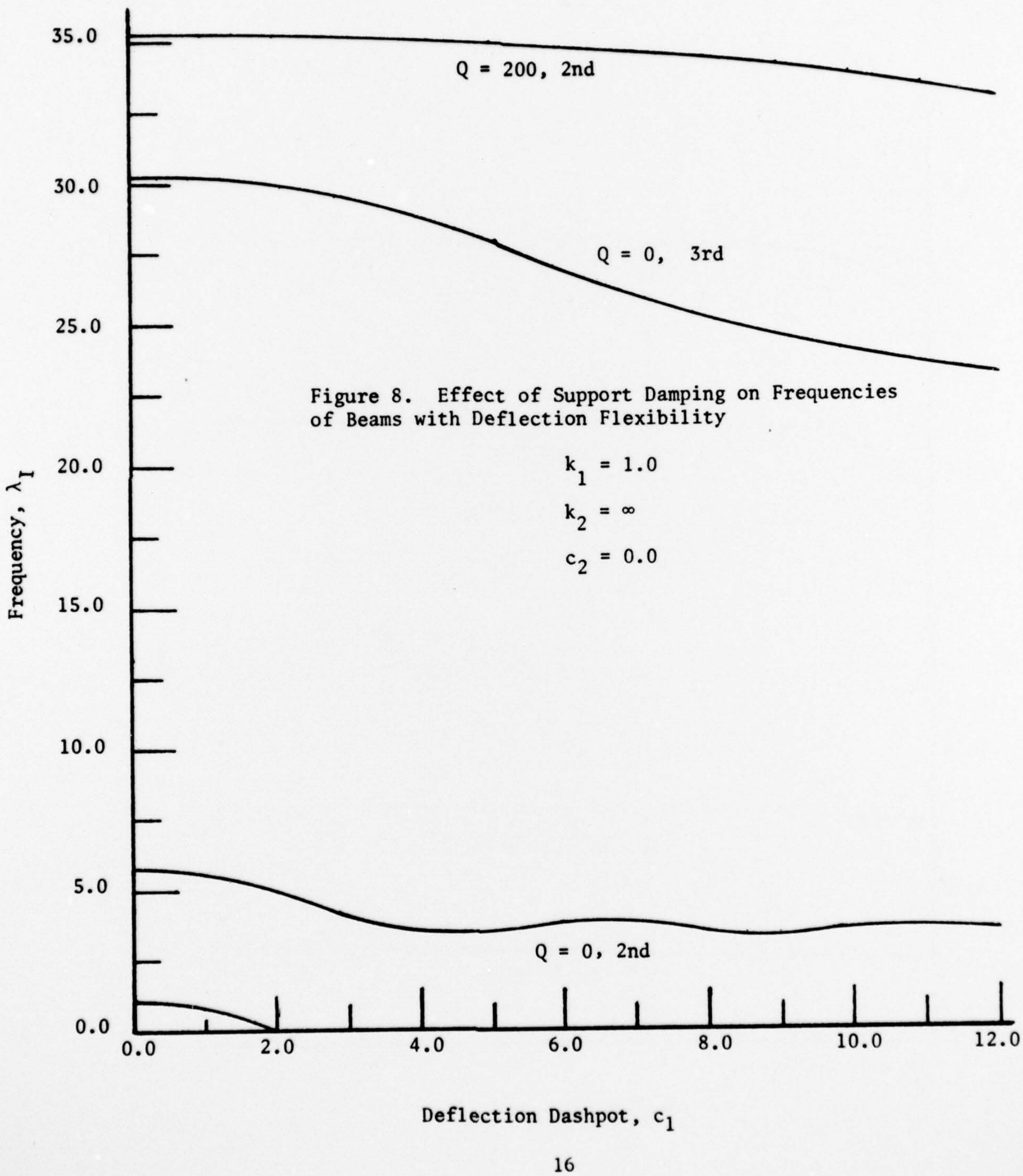


Figure 8. Effect of Support Damping on Frequencies of Beams with Deflection Flexibility

$k_1 = 1.0$

$k_2 = \infty$

$c_2 = 0.0$

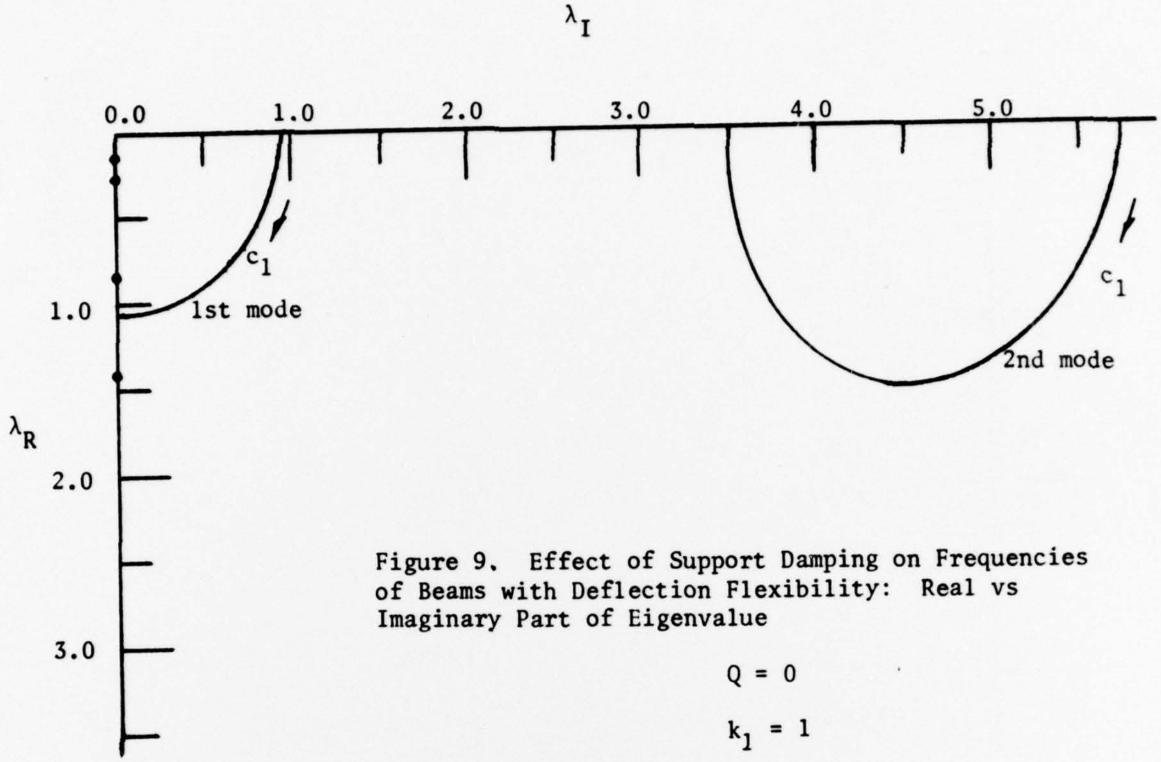


Figure 9. Effect of Support Damping on Frequencies of Beams with Deflection Flexibility: Real vs Imaginary Part of Eigenvalue

$Q = 0$

$k_1 = 1$

$k_2 = \infty$

$c_2 = 0$

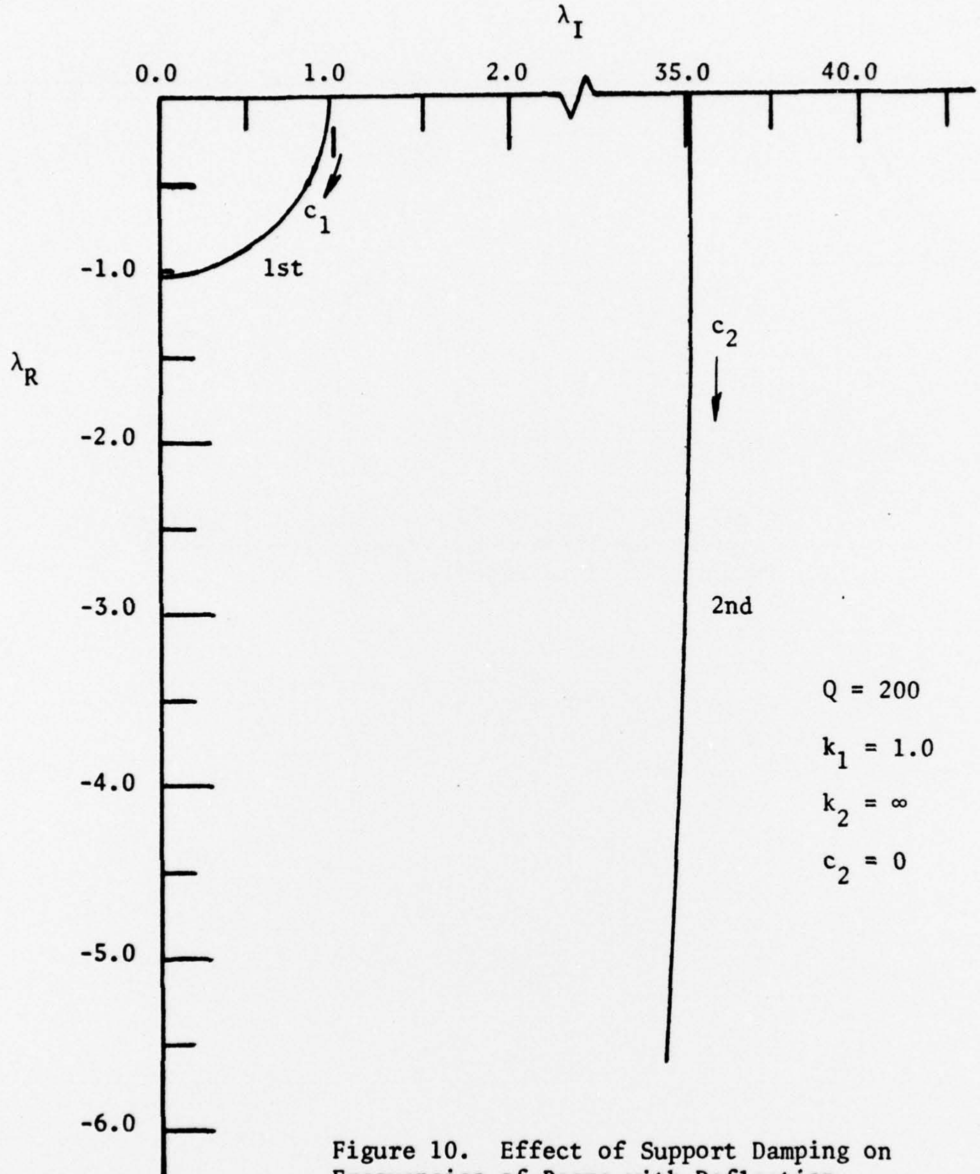


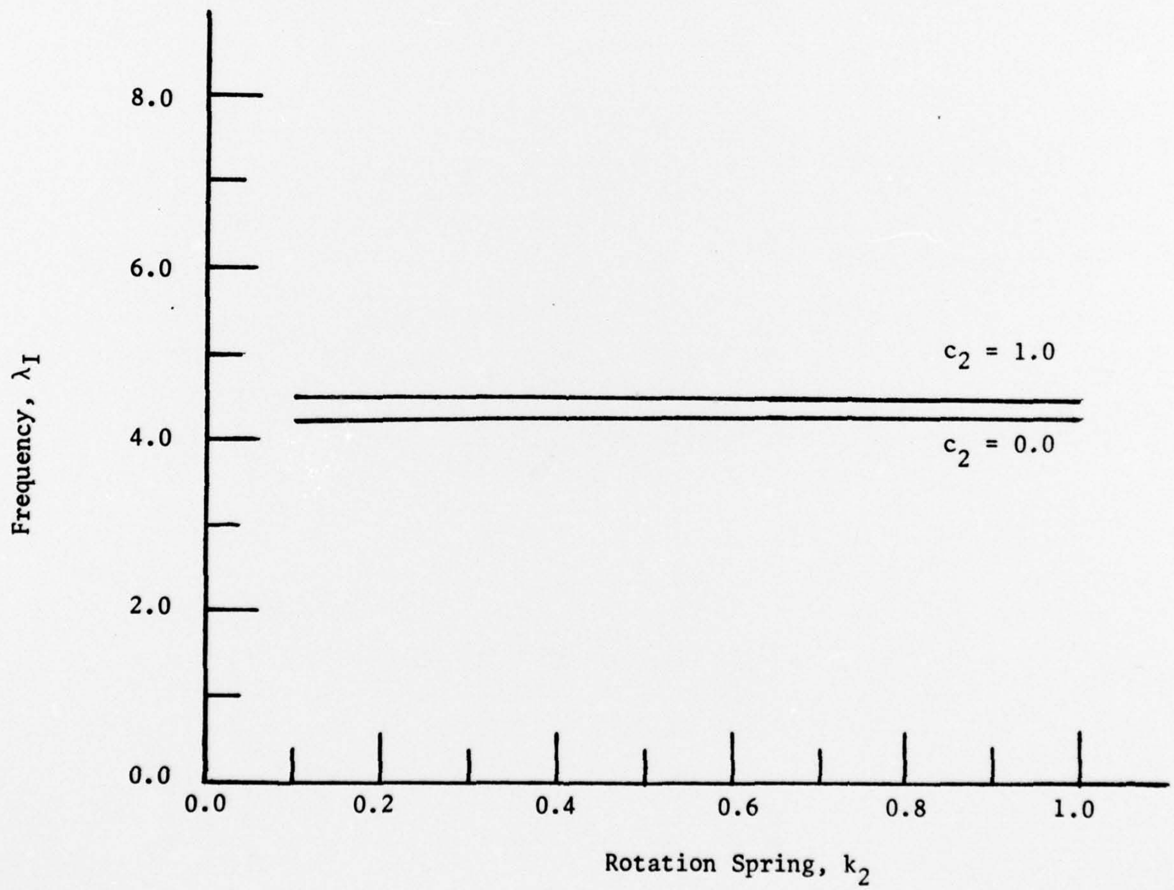
Figure 10. Effect of Support Damping on Frequencies of Beams with Deflection Flexibility: Real vs Imaginary Part of Eigenvalue

Figure 11. Effect of Rotation Spring on Frequencies for Fixed Finite Deflection Spring

$Q = 25.0$

$k_1 = 25.0$

$c_1 = 5.0$



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